General Property of Neutrino Mass Matrix and CP Violation*

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It is found that the atmospheric neutrino mixing angle of θ_{atm} is determined to be $\tan\theta_{atm} = \mathrm{Im}(B)/\mathrm{Im}(C)$ for $B{=}M_{\nu_e\nu_\mu}$ and $C{=}M_{\nu_e\nu_\tau}$, where M_{ij} is the ij element of $M_\nu^\dagger M_\nu$ with M_ν as a complex symmetric neutrino mass matrix in the (ν_e, ν_μ, ν_τ)-basis. Another mixing angle, θ_{13}, defined as $U_{e3} = \sin\theta_{13}e^{-i\delta}$ is subject to the condition: $\tan 2\theta_{13} \propto |\sin\theta_{atm}B + \cos\theta_{atm}C|$ and the CP-violating Dirac phase of δ is identical to the phase of $\sin\theta_{atm}B^* + \cos\theta_{atm}C^*$. The smallest value of $|\sin\theta_{13}|$ is achieved at $\tan\theta_{atm} = -\mathrm{Re}(C)/\mathrm{Re}(B)$ that yields the maximal CP-violation and that implies $C = -\kappa B^*$ for the maximal atmospheric neutrino mixing of $\tan\theta_{atm} = \kappa = \pm 1$. The generic smallness of $|\sin\theta_{13}|$ can be ascribed to the tiny violation of the electron number conservation.

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I. INTRODUCTION

Properties of neutrino oscillations have been extensively studied in various experiments [1, 2] since the confirmation of the atmospheric neutrino oscillations by the SuperKamiokande collaborations in 1998 [3]. It is next expected that leptonic CP-violation can be observed in neutrino-related reactions [4] via CP-violating phases in the neutrino sector [5]. The CP-violating phases are introduced by the PMNS neutrino mixing matrix, U_{PMNS} [6]. The standard parameterization of U_{PMNS} with the CP-violating phases denoted by δ, ρ and σ is given by

$$U_{PMNS} = U_\nu K \tag{1}$$

with

$$\begin{aligned}
U_\nu &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin\theta_{13}e^{i\delta} & 0 & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, && (2) \\
K &= \mathrm{diag}(e^{i\rho}, e^{i\sigma}, 1), && (3)
\end{aligned}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$, if observed neutrino oscillations are induced by the mixing among massive Majorana neutrinos of the three flavors, $\nu_{e,\mu,\tau}$. The recent experimental data [2, 7] show that

$$0.67 \leq \sin^2 2\theta_\odot \leq 0.93, \quad \sin^2 2\theta_{atm} \geq 0.85, \quad |U_{e3}|^2 = \sin^2\theta_{13} < 0.048, \tag{4}$$

where $\theta_\odot = \theta_{12}$ and $\theta_{atm} = \theta_{23}$, and that the magnitudes of three massive neutrino masses, $m_{1,2,3}$, are constrained as

$$\Delta m^2_{atm} = |m_3^2 - m_2^2| = (1.1 - 3.4) \times 10^{-3}\ \mathrm{eV}^2, \quad \Delta m^2_\odot = |m_2^2 - m_1^2| = (5.4 - 9.4) \times 10^{-5}\ \mathrm{eV}^2. \tag{5}$$

These experimental results create mysteries why the observed values are so realized in neutrino oscillations. The presence of tiny masses for neutrinos implied by Eq.(5) is understood by the seesaw mechanism [8, 9] and by the radiative mechanism [10, 11]. It is also stressed that the oscillations show 1) a hierarchy of $\Delta m^2_{atm} \gg \Delta m^2_\odot$ and 2)

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$\sin^2 2\theta_{atm,\odot} = \mathcal{O}(1)$ while $\sin^2\theta_{13} \ll 1$. Among various theoretical proposals to find clues behind the mystery, there are theoretical ideas based on 1) a conservation of the $L_e - L_\mu - L_\tau$ ($\equiv L'$) number [12] and 2) a μ-τ permutation symmetry [13, 14, 15, 16, 17]. The hierarchy of $\Delta m^2_{atm} \gg \Delta m^2_\odot$ is due to the ideal situation with $\Delta m^2_\odot = 0$ and $\Delta m^2_{atm} \neq 0$, which are accounted by the L'-conservation, while the maximal mixing of $\sin^2 2\theta_{atm} = 1$ arises from the μ-τ permutation symmetry. However, to discuss how to depart from these ideal cases is physically important. Furthermore, if CP-violating phases are included, the possible form of the neutrino mass matrix is not fully understood [18]. We would like to discuss it to clarify its general properties and implications on neutrino physics.

II. NEUTRINO MASS MATRIX

The neutrino mass terms are described by

$$-\mathcal{L}_{mass} \ = \ \frac{1}{2}\left(\nu_e, \nu_\mu, \nu_\tau\right)^T M_\nu \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} + \text{h.c.}, \tag{6}$$

where M_ν is a complex symmetric mass matrix. It is understood that the charged leptons and neutrinos are rotated, if necessary, to give diagonal charged-current interactions and to define ν_e, ν_μ and ν_τ. This flavor neutrino mass matrix can be diagonalized by U_{PMNS} to give

$$U^T_{PMNS} M_\nu U_{PMNS} = M^{diag}_\nu = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}. \tag{7}$$

Since M_ν is not Hermitian, one has to deal with the complexity due to the existence of all three phases, one Dirac phase of δ [19] and two Majorana phases of ρ and σ [20]. As a simpler choice, we use a Hermitian matrix [21, 22]:

$$\begin{aligned} &M = M^\dagger_\nu M_\nu, \\ &U^\dagger_{PMNS} M U_{PMNS} = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}, \end{aligned} \tag{8}$$

to examine the structure of M_ν so that two Majorana phases in K become irrelevant. We parameterize M_ν by

$$M_\nu = \begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix}, \tag{9}$$

leading to

$$M = \begin{pmatrix} A & B & C \\ B^* & D & E \\ C^* & E^* & F \end{pmatrix}, \tag{10}$$

where

$$\begin{aligned} &A = |a|^2 + |b|^2 + |c|^2, \quad B = a^* b + b^* d + c^* e, \quad C = a^* c + b^* e + c^* f, \\ &D = |b|^2 + |d|^2 + |e|^2, \quad E = b^* c + d^* e + e^* f, \quad F = |c|^2 + |e|^2 + |f|^2. \end{aligned} \tag{11}$$

Note that B, C and E are complex.

To examine the possible form of M_ν compatible with the observed data of Eqs(4) and (5), we have directly performed the computation of Eq.(8) and have found the following constraints:

$$c_{12}\Delta_1 - s_{12}\left[\tilde{s}^*_{13}\left(c_{23}B^* - s_{23}C^*\right) + c_{13}\Delta_2\right] = 0, \quad s_{12}\Delta_1 + c_{12}\left[\tilde{s}^*_{13}\left(c_{23}B^* - s_{23}C^*\right) + c_{13}\Delta_2\right] = 0, \tag{12}$$

$$c_{12}\left(s_{12}\lambda_1 + c_{12}\left[c_{13}\left(c_{23}B - s_{23}C\right) - \tilde{s}^*_{13}\Delta^*_2\right]\right) - s_{12}\left(c_{12}\lambda_2 + s_{12}\left[c_{13}\left(c_{23}B^* - s_{23}C^*\right) - \tilde{s}_{13}\Delta_2\right]\right) = 0, \tag{13}$$

for $\tilde{s}_{13} = s_{13}e^{i\delta}$, where $\Delta_{1,2}$ are defined to be:

$$\Delta_1 = c_{13}\tilde{s}^*_{13}\left(A - \lambda_3\right) + c^2_{13}\left(s_{23}B + c_{23}C\right) - \tilde{s}^{*2}_{13}\left(s_{23}B^* + c_{23}C^*\right), \quad \Delta_2 = c^2_{23}E - s^2_{23}E^* + s_{23}c_{23}\left(D - F\right), \tag{14}$$

and the diagonalized masses:

$$
\begin{aligned}
& m_1^2 = c_{12}^2\lambda_1 + s_{12}^2\lambda_2 - 2c_{12}s_{12}X, \quad m_2^2 = s_{12}^2\lambda_1 + c_{12}^2\lambda_2 + 2c_{12}s_{12}X, \\
& m_3^2 = c_{13}^2\lambda_3 + s_{13}^2 A + c_{13}\left[\tilde{s}_{13}\left(s_{23}B + c_{23}C\right) + \tilde{s}_{13}^*\left(s_{23}B^* + c_{23}C^*\right)\right],
\end{aligned}
\tag{15}
$$

and

$$
\begin{aligned}
& \lambda_1 = c_{13}^2 A - c_{13}\left[\tilde{s}_{13}\left(s_{23}B + c_{23}C\right) + \tilde{s}_{13}^*\left(s_{23}B^* + c_{23}C^*\right)\right] + s_{13}^2\lambda_3, \\
& \lambda_2 = c_{23}^2 D + s_{23}^2 F - 2s_{23}c_{23}\mathrm{Re}\left(E\right), \quad \lambda_3 = s_{23}^2 D + c_{23}^2 F + s_{23}c_{23}\mathrm{Re}\left(E\right). \\
& 2X = c_{13}\left(c_{23}B - s_{23}C\right) - \tilde{s}_{13}^*\Delta_2^* + c_{13}\left(c_{23}B^* - s_{23}C^*\right) - \tilde{s}_{13}\Delta_2,
\end{aligned}
\tag{16}
$$

Since Eq.(12) gives

$$
\Delta_1 = 0, \quad \tilde{s}_{13}^*\left(c_{23}B^* - s_{23}C^*\right) + c_{13}\Delta_2 = 0,
\tag{17}
$$

we obtain that

$$
\tan 2\theta_{13} = 2\frac{\left|s_{23}B + c_{23}C\right|}{\lambda_3 - A}, \quad X = \frac{c_{23}\mathrm{Re}\left(B\right) - s_{23}\mathrm{Re}\left(C\right)}{c_{13}},
\tag{18}
$$

and δ used in $\tilde{s}_{13}$ is determined by the phase of $s_{23}B + c_{23}C$ to be:

$$
s_{23}B + c_{23}C = \left|s_{23}B + c_{23}C\right| e^{-i\delta},
\tag{19}
$$

provided that $s_{23}B + c_{23}C \neq 0$. By using Eq.(13) with Eq.(17) for Δ_2, we find that

$$
\tan\theta_{23} = \frac{\mathrm{Im}\left(B\right)}{\mathrm{Im}\left(C\right)},
\tag{20}
$$

from the constraint on the imaginary part:

$$
c_{23}B - s_{23}C = c_{23}B^* - s_{23}C^*,
\tag{21}
$$

and

$$
\tan 2\theta_{12} = 2\frac{X}{\lambda_2 - \lambda_1},
\tag{22}
$$

from the constraint on the real part. As a result, δ is expressed as:

$$
\tan\delta = -\frac{1}{s_{23}}\frac{\mathrm{Im}\left(B\right)}{s_{23}\mathrm{Re}\left(B\right) + c_{23}\mathrm{Re}\left(C\right)}.
\tag{23}
$$

Furthermore, considering the relations of Δ_2 in Eqs.(14) and (17), we also obtain that

$$
\begin{aligned}
& \mathrm{Im}\left(E\right) = s_{13}X\sin\delta, \\
& \cos 2\theta_{23}\mathrm{Re}\left(E\right) = \frac{\sin 2\theta_{23}}{2}\left(F - D\right) - s_{13}X\cos\delta.
\end{aligned}
\tag{24}
$$

From these relations, the mass parameters are further converted to give

$$
m_1^2 = \frac{\lambda_1 + \lambda_2}{2} - \frac{X}{\sin 2\theta_{12}}, \quad m_2^2 = \frac{\lambda_1 + \lambda_2}{2} + \frac{X}{\sin 2\theta_{12}}, \quad m_3^2 = \frac{c_{13}^2\lambda_3 - s_{13}^2 A}{c_{13}^2 - s_{13}^2},
\tag{25}
$$

where

$$
\lambda_1 = \frac{c_{13}^2 A - s_{13}^2\lambda_3}{c_{13}^2 - s_{13}^2},
\tag{26}
$$

together with Eq.(16) for $\lambda_{2,3}$. We find that

$$
\Delta m_\odot^2 = 2\frac{c_{23}\mathrm{Re}\left(B\right) - s_{23}\mathrm{Re}\left(C\right)}{c_{13}\sin 2\theta_{12}},
\tag{27}
$$

which is the useful relation.

If other parameterizations of the CP-violating Dirac phase in U_{PMNS} are employed, different relations will be derived. However, this difference can be absorbed in the redefinition of the masses. For example, results from U_{PMNS} of the Kobayashi-Maskawa type (with $e^{-i\delta}$ as a CP-violating phase) [23] can be generated by the replacement of B and C by $Be^{-i\delta}$ and $Ce^{-i\delta}$ in the same M as Eq.(10) with the standard U_{PMNS} of Eq.(2). Especially, in Eq.(19), $s_{23}B + c_{23}C = |s_{23}B + c_{23}C|\, e^{-i\delta}$ becomes $s_{23}Be^{-i\delta} + c_{23}Ce^{-i\delta} = |s_{23}B + c_{23}C|\, e^{-i\delta}$, leading to $s_{23}B + c_{23}C = |s_{23}B + c_{23}C|$(=real). Therefore, no CP-violating phase is induced by Eq.(19). Instead, the CP-violating phase is induced by $c_{23}B - s_{23}C = |c_{23}B - s_{23}C|e^{i\delta}$ derived as a solution of $(s_{23}B - c_{23}C)e^{-i\delta}$=$(s_{23}B^* - c_{23}C^*)e^{i\delta}$ from Eq.(21) with the appropriate replacement of B and C. Since $s_{23}B + c_{23}C$=real, the relation of $\tan\theta_{23}$ becomes $\tan\theta_{23} = -\mathrm{Im}(C)/\mathrm{Im}(B)$ instead of Eq.(20). In this article, we only show the results by using the standard U_{PMNS}.

Since $\sin^2\theta_{13} < 0.048$ is reported, let us choose $\sin\theta_{13} = 0$ and no CP-violation phase is induced by Eq.(19) because of $s_{23}B + c_{23}C = 0$ in Eq.(18), leading to real B and C from Eq.(21). The mixing angle of θ_{23} is determined by [16]

$$\tan\theta_{23} = -\frac{C}{B}, \tag{28}$$

which should be compared with Eq.(20) for the complex B and C. We obtain that $\mathrm{Im}(E) = 0$ and

$$2E\cos 2\theta_{23} = (F - D)\sin 2\theta_{23}, \tag{29}$$

from Eq.(24) with $t_{13} = 0$. The solar neutrino mixing angle of θ_{12} is given by

$$\tan 2\theta_{12} = \frac{2B}{c_{23}\left(\lambda_2 - \lambda_1\right)}. \tag{30}$$

The observed atmospheric neutrino mixing is close to the maximal one. We, then, restrict ourselves to the case with $\tan\theta_{23} = \pm 1$ ($\equiv \kappa$) and $\cos\theta_{23} > 0$. The relation in Eq.(20) becomes

$$\mathrm{Im}\left(C\right) = \kappa \mathrm{Im}\left(B\right), \tag{31}$$

and the constranint on E becomes

$$\kappa\left(F - D\right) = \sqrt{2}t_{13}\cos\delta\left(\mathrm{Re}\left(B\right) - \kappa\mathrm{Re}\left(C\right)\right), \quad \sqrt{2}\mathrm{Im}\left(E\right) = t_{13}\sin\delta\left(\mathrm{Re}\left(B\right) - \kappa\mathrm{Re}\left(C\right)\right). \tag{32}$$

If no CP-violation exists, $\delta = 0$ is required. Namely, it demands $\mathrm{Im}(B) = 0$ from Eq.(23), which, in turn, gives $\tan\theta_{23} = 0$ if $\mathrm{Im}(C) \neq 0$. To get around $\tan\theta_{23} = 0$, $\mathrm{Im}(C) = 0$ should be imposed and Eq.(31) disappears. As a result, B, C and E turn out to be all real. The mixing angle of θ_{23} is determined by κ derived from Eq.(32) with $\delta = 0$. Therefore, in order to ensure the appearance of the maximal atmospheric neutrino mixing, we have to be careful to impose the constraint:

- $\mathrm{Im}\left(C\right) = \kappa\mathrm{Im}\left(B\right)$ for the presence of CP-violation,
- $\kappa\left(F - D\right) = \sqrt{2}t_{13}(B - \kappa C)$ with real B and C for the absence of CP-violation.

It should be noted that, for the absence of CP-violation, the approximate equality of $F \sim D$ is often assumed and it results in the familiar relation of $\tan\theta_{23}(=\kappa) \sim -C/B$ for $|t_{13}| \sim 0$. Another solution with $\tan\theta_{23}(=\kappa) \sim B/C$ seems appropriate because it may yield $\sin^2 2\theta_{12} \sim 0$ in Eq.(22).

III. EXAMPLES

To see how the present analysis works, let us examine the specific example of M_ν with $c = -\kappa b^*$ given by

$$M_\nu = \left(\begin{array}{ccc} a & b & -\kappa b^* \\ b & d & e \\ -\kappa b^* & e & d^* \end{array} \right), \tag{33}$$

whose physical consequence has been discussed in Ref.[24]. From $M_\nu^\dagger M_\nu$, we obtain that

$$\begin{aligned} &A = |a|^2 + 2|b|^2, \quad B = \left(a^* - \kappa e\right)b + b^* d, \quad C = \left(-\kappa a^* + e\right)b^* - \kappa b d^*, \\ &D = F = |b|^2 + |d|^2 + |e|^2, \quad E = -\kappa b^* b^* + 2d^*\mathrm{Re}\left(e\right). \end{aligned} \tag{34}$$

To meet the maximal atmospheric neutrino mixing, one simple choice of mass terms is to assume that a and e are real so that $C = -\kappa B^*$, leading to $\mathrm{Re}(C) = -\kappa\mathrm{Re}(B)$ and $\mathrm{Im}(C) = \kappa\mathrm{Im}(B)$. Therefore, $\tan\theta_{23} = \kappa$ is recovered. From Eq.(23), we have

$$\tan\delta = -\frac{2\mathrm{Im}\,(B)}{\mathrm{Re}\,(B) + \kappa\mathrm{Re}\,(C)} = \pm\infty, \tag{35}$$

leading to $|\delta| = \pi/2$, which indicates the maximal CP-violation [24]. From Eq.(32), we also have $F = D$ by $\cos\delta = 0$ consistent with Eq.(34) as expected and $\mathrm{Im}(E) = \pm\sqrt{2}|t_{13}\mathrm{Re}(B)|$ as an additional constraint, where $\pm$ depends on the sign of δ. Other mixing angles are computed to be:

$$\tan 2\theta_{12} = 2\sqrt{2}\frac{\mathrm{Re}\,(B)}{c_{13}\,(D - \kappa\mathrm{Re}(E) - \lambda_1)}, \quad \tan 2\theta_{13} = 2\sqrt{2}\frac{\mathrm{Im}\,(B)}{D + \kappa\mathrm{Re}(E) - A}. \tag{36}$$

Another example is M_ν with a μ-τ permutation symmetry [13, 14, 15, 16, 17], which suggests that $c = \kappa b$ and $d = f$ giving

$$M_\nu = \begin{pmatrix} a & b & \kappa b \\ b & d & e \\ \kappa b & e & d \end{pmatrix}. \tag{37}$$

From this matrix,

$$\begin{aligned} &A = |a|^2 + 2|b|^2, \quad B = a^*b + b^*d + \kappa b^*e, \quad C = \kappa a^*b + b^*e + \kappa b^*d, \\ &D = F = |b|^2 + |d|^2 + |e|^2, \quad E = \kappa|b|^2 + d^*e + e^*d, \end{aligned} \tag{38}$$

are obtained. Since $C = \kappa B$ and $X = 0$, we find that $\tan\theta_{23} = \kappa$ and $\tan 2\theta_{12} = 0$ unless $\lambda_1 = \lambda_2$. If $\lambda_1 = \lambda_2$, θ_{12} is not fixed and the masses of $m_{1,2}$ turn out to satisfy $m_1^2 = m_2^2$ and $\Delta m^2_\odot = 0$. The mixing angle of θ_{13} and the CP-violating phase of δ are determined to be:

$$\tan 2\theta_{13} = 2\sqrt{2}\frac{|B|}{D + \kappa E - A}, \quad \tan\delta = -\frac{\mathrm{Im}\,(B)}{\mathrm{Re}\,(B)}. \tag{39}$$

Since $\sin^2 2\theta_{12} = 4/(4 + x^2)$ with $x = (\lambda_2 - \lambda_1)/X$, we may require that $x \sim 1$:

$$\lambda_2 - \lambda_1 \sim X(= (B - \kappa C)/c_{13}), \tag{40}$$

that gives $\sin^2 2\theta_{12} \sim 0.8$ after the μ-τ symmetry is slightly at least broken by $\mathrm{Re}(B) \neq \mathrm{Re}(C)$, which also induces $\Delta m^2_\odot \neq 0$.

Finally, let us examine a typical texture among mass matrices with two texture zeros [25], which is given by

$$M_\nu = \begin{pmatrix} 0 & b & 0 \\ b & d & e(= \kappa d) \\ 0 & e(= \kappa d) & f \end{pmatrix}, \tag{41}$$

where the similar matrix with $b = 0$ and $c \neq 0$ can be treated in the same way, and it yields

$$\begin{aligned} &A = |b|^2, \quad B = b^*d, \quad C = b^*e, \\ &D = |b|^2 + |d|^2 + |e|^2, \quad E = d^*e + e^*f, \quad F = |e|^2 + |f|^2. \end{aligned} \tag{42}$$

The simplest way to get the maximal atmospheric neutrino mixing is to require that $e = \kappa d$, leading to $C = \kappa B$ and $\tan\theta_{23} = \kappa$. Then, the same relations for $\tan\theta_{13}$ and $\tan\delta$ as those in the μ-τ symmetric case are also satisfied and, accordingly, θ_{12} is left undetermined. Additional constraints are given by $\arg(d) = \arg(f)$ from $\mathrm{Im}(E) = 0$ and $|f|^2 = |b|^2 + |d|^2$ from $F = D$ imposed by Eq.(32).

IV. DISCUSSIONS

In summary, if CP-violation is present, using the complex symmetric mass matrix given by

$$M_\nu = \begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix}, \tag{43}$$

we have found the important and simple relation:

$$\tan\theta_{23} = \frac{\mathrm{Im}\,(B)}{\mathrm{Im}\,(C)}, \tag{44}$$

for $B = a^*b + b^*d + c^*e$ and $C = a^*c + b^*e + c^*f$. Any models with CP-violation should respect $|\mathrm{Im}\,(B)\,| \sim |\mathrm{Im}\,(C)\,|$ to explain the observed result of $\sin^2 2\theta_{23} \sim 1$. Furthermore, the CP-violating Dirac phase is induced by the phase of $s_{23}B + c_{23}C$. The physically interesting quantity of θ_{13} is determined to be:

$$\tan 2\theta_{13} = \frac{2|s_{23}B + c_{23}C|}{s_{23}^2 D + c_{23}^2 F + 2s_{23}c_{23}\mathrm{Re}\,(E) - A}, \tag{45}$$

for $A = |a|^2 + |b|^2 + |c|^2$, $D = |b|^2 + |d|^2 + |e|^2$, $E = b^*c + d^*e + e^*f$ and $F = |c|^2 + |e|^2 + |f|^2$. Other results include

$$\Delta m^2_\odot = \frac{2X}{\sin 2\theta_{12}}, \quad \tan 2\theta_{12} = \frac{2X}{\lambda_2 - \lambda_1}, \tag{46}$$

where

$$X = \frac{c_{23}\mathrm{Re}\,(B) - s_{23}\mathrm{Re}\,(C)}{\cos\theta_{13}}. \tag{47}$$

In order to understand the origin of $\sin^2\theta_{13} \ll 1$, it is instructive to notice that the mass terms are grouped into three categories according to the electron number L_e [26, 27]. Namely, a has $L_e = 2$, b and c have $L_e = 1$ and d, e and f have $L_e = 0$. If the mass terms with $L_e \neq 0$ are created by perturbative interactions with $|L_e| = 1$, we may assume that $|a| \ll |b, c| \ll |d, e, f|$, leading to $|A| \ll |B, C| \ll |D, E, F|$. Then, this hierarchy ensures the appearance of $|\tan 2\theta_{13}| \ll 1$ due to the tiny violation of the L_e-conservation and at the same time may allow $\Delta m^2_{atm} \gg \Delta m^2_\odot$ to arise if $\Delta m^2_{atm} \gtrsim \mathcal{O}(|D, E, F|)$ since $\Delta m^2_\odot = \mathcal{O}(|\mathrm{Re}(B, C)|)$ [27].

The smallest $|\tan 2\theta_{13}|$ can be acheived at $\tan\theta_{23} = -\mathrm{Re}(C)/\mathrm{Re}(B)$ to yield $|\mathrm{Im}(B)/s_{23}|$, pointing to $\tan\delta = \infty$ from Eq.(23) and showing the maximal CP-violation. Considering $\tan\theta_{23} = \mathrm{Im}(B)/\mathrm{Im}(C)$, we observe that B and C are consistently related to be, for the maximal atmospheric neutrino mixing with $\tan\theta_{23} = \kappa(= \pm 1)$,

$$C = -\kappa B^*, \tag{48}$$

which is the case of Eq.(33) and also corresponds to the mass matrix in Ref.[22]. The same result of the maximal CP-violation is obtained for U_{PMNS} of the Kobayashi-Maskawa type if $\tan\theta_{23} = \mathrm{Re}(B)/\mathrm{Re}(C)$ is chosen and $C = \kappa B^*$ becomes a consistent relation.

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[1] Q.R. Ahmad *et al.* [SNO Collaboration], Phys. Rev. Lett. **87** (2001) 071301; Phys. Rev. Lett. **89** (2002) 011301; S.H. Ahn *et al.* [K2K Collaboration], Phys. Lett. B **511** (2001) 178; Phys. Rev. Lett. **90** (2003) 041801; K. Eguchi *et al.* [KamLAND collaboration], Phys. Rev. Lett. **90** (2003) 021802. See also, C. McGrew, Talk given at *ICHEP 2004: The 32nd International Conference on High Energy Physics*, Beijin, China (Aug. 16-22, 2004); Y. Wang, Talk given at *ICHEP 2004: The 32nd International Conference on High Energy Physics*, Beijin, China (Aug. 16-22, 2004).

[2] M. Apollonio *et al.* [CHOOZ Collaboration], Euro. Phys. J. C **27** (2003) 331; E. Kearns *et al.* [SuperKamiokande Collaboration], Talk given at *Neutrino 2004: The 21st International Conference on Neutrino Physics and Astrophysics*, Paris, France (June 14-19, 2004); T. Nakaya *et al.* [K2K Collaboration], Talk given at *Neutrino 2004: The 21st International Conference on Neutrino Physics and Astrophysics*, Paris, France (June 14-19, 2004); T. Araki *et al.* [KamLAND Collaboration], "Measurement of Neutrino Oscillation with KamLAND: Evidence of Spectral Distortion", [arXive:hep-ex/0406035].

[3] Y. Fukuda *et al.* [Super-Kamiokande Collaboration], Phys. Rev. Lett. **81** (1998) 1158 [Erratum-ibid **81** (1998) 4297]; Phys. Rev. Lett. **82** (1999) 2430. See also, T. Kajita and Y. Totsuka, Rev. Mod. Phys. **73** (2001) 85.

[4] See for example, A. Blondel, Talk given at *Neutrino 2004: The 21st International Conference on Neutrino Physics and Astrophysics*, Paris, France (June 14-19, 2004); H. Sugiyama, Talk given at *ICHEP 2004: The 32nd International Conference on High Energy Physics*, Beijin, China (Aug. 16-22, 2004). See also, J. Burguet-Castell, M.B. Gavela, J.J. Gomez-Cadenas, P. Hernandez and O. Mena, Nucl. Phys. B **646** (2002) 301; W.J. Marciano, "Extra Long Baseline Neutrino Oscillations and CP Violation", [arXive:hep-ph/0108181]; J. Burguet Castell and O. Mena, Nucl. Instrum. Meth A **503** (2003) 199; T. Ota and J. Sato, Phys. Rev. D **67** (2003) 053003; T. Ota, J. Phys. G **29** (2003) 1869; M.V. Diwan, Int. J. Mod. Phys. A **18** (2003) 4039; H. Minakata and H. Sugiyama, Phys. Lett. B **580** (2004) 216.

[5] S.M. Bilenky, J. Hosek and S.T. Petcov, Phys. Lett. **94B** (1980) 495; J. Schechter and J.W.F. Valle, Phys. Rev. D **22** (1980) 2227; M. Doi, T. Kotani, H. Nishiura, K. Okuda and E. Takasugi, Phys. Lett. **102B** (1981) 323.

[6] B. Pontecorvo, JETP (USSR) **34** (1958) 247; Zh. Eksp. Teor. Piz. **53** (1967) 1717; Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28** (1962) 870.

[7] See for example, S. Goswami, Talk given at *Neutrino 2004: The 21st International Conference on Neutrino Physics and Astrophysics*, Paris, France (June 14-19, 2004); G. Altarelli, Talk given at *Neutrino 2004: The 21st International Conference on Neutrino Physics and Astrophysics*, Paris, France (June 14-19, 2004); A. Bandyopadhyay, "Update of the solar neutrino oscillation analysis with the 766 Ty KamLAND spectrum", [arXive:hep-ph/0406328].

[8] T. Yanagida, in *Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe* edited by A. Sawada and A. Sugamoto (KEK Report No.79-18, Tsukuba, 1979), p.95; Prog. Theor. Phys. **64** (1980) 1103; M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity* edited by P. van Nieuwenhuizen and D.Z. Freedmann (North-Holland, Amsterdam 1979), p.315; R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44** (1980) 912. See also, P. Minkowski, Phys. Lett. **B67** (1977) 421.

[9] R.N. Mohapatra and G. Senjanović, Phys. Rev. D **23** (1981) 165; C. Wetterich, Nucl. Phys. B **187** (1981) 343. See also, J. Schechter and J.W.F. Valle, in Ref.[5].

[10] A. Zee, Phys. Lett. **93B** (1980) 389; Phys. Lett. **161B** (1985) 141; L. Wolfenstein, Nucl. Phys. B **175** (1980) 93, S. T. Petcov, Phys. Lett. **115B** (1982) 401.

[11] A. Zee, Nucl. Phys. **264B** (1986) 99; K. S. Babu, Phys. Lett. B **203** (1988) 132; D. Chang, W-Y. Keung and P.B. Pal, Phys. Rev. Lett. **61** (1988) 2420; J. Schechter and J.W.F. Valle, Phys. Lett. B **286** (1992) 321.

[12] R. Barbieri, L.J. Hall, D. Smith, N.J. Weiner and A. Strumia, JHEP **12** (1998) 017. See also S.T. Petcov, Phys. Lett. **110B** (1982) 245; C.N. Leung and S.T. Petcov, Phys. Lett. **125B** (1983) 461; A. Zee, Phys. Lett. **161B** (1985) 141.

[13] T. Fukuyama and H. Nishiura, in *Proceedings of International Workshop on Masses and Mixings of Quarks and Leptons* edited by Y. Koide (World Scientific, Singapore, 1997), p.252; "Mass Matrix of Majorana Neutrinos", [arXive:hep-ph/9702253]; Y. Koide, H. Nishiura, K. Matsuda, T. Kikuchi and T. Fukuyama, Phys. Rev. D **66** (2002) 093006; Y. Koide, Phys. Rev. D **69** (2004) 093001; K. Matsuda and H. Nishiura, Phys. Rev. D **69** (2004) 117302.

[14] E. Ma, M. Raidal and U. Sarkar, Phys. Rev. Lett. **85** (2000) 3769; S.K. Kang and C.S. Kim, Phys. Rev. D **63** (2001) 113010; I. Dorsner and S.M. Barr, Nucl. Phys. B **617** (2001) 493.

[15] W. Grimus and L. Lavoura, JHEP **0107** (2001) 045; Euro. Phys. J. C **28** (2003) 123; Phys. Lett. B **573** (2003) 189; J. Phys. G **30** (2004) 73; P.F. Harrison and W.G. Scott, Phys. Lett. B **547** (2002) 219.

[16] T. Kitabayashi and M. Yasuè, Phys. Lett. B **524** (2002) 308; Int. J. Mod. Phys. A **17** (2002) 2519; Phys. Rev. D **67** (2003) 015006; I. Aizawa, M. Ishiguro, T. Kitabayashi and M. Yasuè, Phys. Rev. D **70** (2004) 015011.

[17] R.N. Mohapatra, JHEP **0410** (2004) 027; W. Grimus, A.S. Joshipura, S. Kaneko, L. Lavoura, H. Sawanaka and M. Tanimoto, "Non-Vanishing U_{e3} and $\cos 2\theta_{23}$ from a broken Z_2 symmetry", [arXive:hep-ph/0408123].

[18] See for example, M. Frigerio and A.Yu. Smirnov, Nucl. Phys. B **640** (2002) 233; Phys. Rev. D **67** (2003) 013007; S.F. King, in *Proceedings of 10th International Workshop on Neutrino Telescopes* edited by M. Baldo-Ceolin (U. of Padua Publication, Italy, 2003), "Neutrino Mass, Flavor and CP Violation", [arXive:hep-ph/0306095]; Z.Z. Xing, Int. J. Mod. Phys. A **19** (2004) 1; O.L.G. Peres and A.Yu. Smirnov, Nucl. Phys. B **680** (2004) 479; C.H. Albright, Phys. Lett. B **599** (2004) 285; J. Ferrandis and S. Pakvasa, Phys. Lett. B **603** (2004) 184; S. Zhou and Z.Z. Xing, "A Systematic Study of Neutrino Mixing and CP Violation from Lepton Mass Matrices with Six Texture Zeros", [arXive:hep-ph/0404188]; S.T. Petcov and W. Rodejohann, "Flavor Symmetry $L_e - L_\mu - L_\tau$, Atmospheric Neutrino Mixing and CP Violation in the Lepton Sector", [arXive:hep-ph/0409135].

[19] P.I. Krastev and S.T. Petcov, Phys. Lett. B **205** (1988) 84.

[20] S.M. Bilenky, S. Pascoli and S.T. Petcov, Phys. Rev. D **64** (2001) 053010.

[21] See for example, P.F. Harrison, D.H. Perkins and W.G. Scott, Phys. Lett. B **530** (2002) 167; Z.Z. Xing, Phys. Lett. B **533** (2002) 85; P.F. Harrison and W.G. Scott, Phys. Lett. B **557** (2003) 76.

[22] P.F. Harrison and W.G. Scott, Phys. Lett. B **594** (2004) 324.

[23] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49** (1973) 652.

[24] E. Ma, Mod. Phys. Lett. A **17** (2002) 2361; K.S. Babu, E. Ma and J.W.F. Valle, Phys. Lett. B **552** (2003) 207; W. Grimus and L. Lavoura, Phys. Lett. B **579** (2004) 113. See also Ref.[22].

[25] P.H. Frampton, S.L. Glashow and D. Marfatia, Phys. Lett. B **536** (2002) 79; Z.Z. Xing, Phys. Lett. B **530** (2002) 159.

[26] See for example, M. Frigerio and A.Yu. Smirnov, in Ref.[18].

[27] I. Aizawa, M. Ishiguro, T. Kitabayashi and M. Yasuè, in Ref.[16].